

LABORATORY BOUNDS ON LORENTZ SYMMETRY VIOLATION IN LOW ENERGY NEUTRINO PHYSICS*

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Quantitative bounds on Lorentz symmetry violation in the neutrino sector have been obtained by analyzing existing laboratory data on neutron β decay and pion leptonic decays. In particular some parameters appearing in the energy-momentum dispersion relations for ν_e and ν_μ have been constrained in two typical cases arising in many models accounting for Lorentz violation.

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I. LORENTZ-VIOLATING DISPERSION RELATIONS

In recent times ultra-high energy Lorentz symmetry violations have been investigated, both theoretically and experimentally,¹ by means of quite different approaches, sometimes extending, sometimes abandoning the formal and conceptual framework of Einstein’s Special Relativity. The most important consequence of a Lorentz violation (LV) is the modification of the ordinary momentum-energy dispersion law $E^2 = \mathbf{p}^2 + m^2$ at energy scales usually assumed of the order of the Planck mass, by means of additional terms which vanish in the low momentum limit. Lorentz-breaking observable effects appear in Grand-Unification Theories [1], in (Super)String/Brane theories [2], in Quantum Gravity [3], in foam-like quantum spacetimes [4]; in spacetimes endowed with a nontrivial topology or with a discrete structure at the Planck length [5,6], or with a (canonical or noncanonical) noncommutative geometry [7–9]; in the so-called “extensions” of the Standard Model incorporating breaking of Lorentz and CPT symmetries [10]; in theories with a variable speed of light or variable physical constants. In particular, the M-Theory [2] and the Loop Quantum Gravity (as in the version with semiclassical spin-network structure of spacetime) [5,6,11] lead to postulate an essentially discrete, quantized spacetime, where a fundamental mass-energy scale naturally arises, in addition to \hbar and c . An intrinsic length is directly correlated to the existence of a “cut-off” in the transferred momentum necessary to avoid the occurrence of “UV catastrophes” in Quantum Field Theories. Other divergences, as those emerging in black hole entropy, could possibly be averted in the presence of quantum LV’s.

A natural extension of the standard dispersion law can be put in most cases under the general form ($p \equiv |\mathbf{p}|$)

$$E^2 = p^2 + m^2 + p^2 f(p/M), \quad (1)$$

where M indicates a (large) mass scale characterizing LV. By using a series expansion for f , under the assumption being M a very large quantity, we can consider only the lower order nonzero term in the expansion:

$$E^2 = p^2 + m^2 + \alpha p^2 \left(\frac{p}{M} \right)^n. \quad (2)$$

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¹Hereafter for simplicity we use the term “violation” or “breakdown” of the Lorentz symmetry, but, in some theories, although Special Relativity does not hold anymore, an underlying extended Lorentz invariance exists (this happens, for example, in “Doubly” Special Relativity [7,8], where “deformed” 4-rotation generators are considered).

The most recurring exponent in the literature on LV is $n = 1$:

$$E^2 = p^2 + m^2 + \alpha \frac{p^3}{M}. \quad (3)$$

Indeed in “noncritical”-Liouville String Theory [12], we have $E^2 = p^2 + m^2 + \xi g_s p^3/M_s$, where g_s is the string coupling and M_s is a string (non-Planck) mass scale; similar expressions are obtained in the Standard Model extensions of Colladay and Kostelecký, as well as in theories with a spacetime “medium” or quantum foam, or even in Quantum Gravity. We find dispersion laws analogous to the $n = 1$ case in Deformed or Doubly Special Relativity, [8,9,13] working in k-deformed Lie-algebra noncommutative (k-Minkowski) spacetimes, in which both the Planck scale and the speed of light act as characteristic scales of a 6-parameter group of spacetime 4-rotations with deformed but preserved Lorentz symmetries. For example, in [8] where the Lie algebra is given by $[x_i, x_0] = ilx_i$, $[x_i, x_k] = 0$ (l being a very small length which may eventually be taken of the order of M_{Planck}^{-1}) we have

$$\frac{e^{lE} + e^{-lE} - 2}{l^2} - p^2 e^{-lE} = m^2 \quad (4)$$

which for $p \gg m$ and considering terms up to $O(l^2)$ reduces to the above seen cubic form

$$E^2 = p^2 + m^2 \mp lp^3. \quad (5)$$

The $n = 0$ case in (1) has been studied by Coleman and Glashow [14], while $n = 2$ is found for some “Quantum Deformed Poincaré Groups” [9], in spacetime foam scenarios at low energies [4,12] and in effective field theories including Lorentz violating dimension-5 operators [15,16]. Different dispersion laws arise from canonical Loop Gravity, Supergravity, or String Theory critical in B-background [17,18], and from canonical noncommutative geometries [8]. For example in [8], assuming the canonical commutation relation $[x^\mu, x^\nu] = i\theta^{\mu\nu}$ we find

$$E^2 = p^2 + m^2 + \frac{\eta}{(p_\mu \theta^{\mu\nu})^2}. \quad (6)$$

Other important models, which assume the Planck 4-momentum (a new Lorentz invariant besides the speed of light) as a frame-independent quantity, entail a linear behavior (i.e., $n = -1$): in [6,9,19–21] for $p \gg m$ and up to terms of the order of μ^2 , μ driving the mass scale of the Lorentz breakdown, we have

$$E^2 = p^2 + m^2 \mp 2\mu p. \quad (7)$$

In such models it is interesting to study the (small momenta) nonrelativistic limit wherein the nonstandard linear term rules.

As pointed out by Bertolami [22], evidences of violation of the Lorentz symmetry seem to emerge from the observation of: a) ultra-high energy cosmic rays with energies [23] beyond the Greisen-Zatsepin-Kuzmin [24] cut-off (of the order of 4×10^{19} eV); b) gamma rays with energies beyond 20 TeV from distant sources such as Markarian 421 and Markarian 501 blazars [25]; c) longitudinal evolution of air showers produced by ultra-high energy hadronic particles which seem to suggest that pions are more stable than expected [26].

The theoretical applications of the modified dispersion relation do carry many threshold effects associated to asymmetric momenta in photoproduction, pair creation, photon stability, vacuum Čerenkov effects, etc. [27,28], long baseline dispersion and vacuum birefringence (signals from gamma ray bursts, active galactic nuclei, pulsars) [29], dynamical effects of LV background fields (gravitational coupling and additional wave modes) [15], different maximum speeds for different particles [14,15].

An interesting consequence of a nonstandard dispersion law is an essentially “non-Newtonian” dynamics at very high energies.² All that recalls the non-Newtonian features of spinning particles dynamics at the Compton scale \hbar/m [30].

There are further deep experimental implications at low energy [10,31,32], mostly in the Standard Model extensions based on a Lagrangian containing any possible phenomenologically relevant Lorentz- and CPT-violating term, for hadrons [33], nucleons [34], electrons [35], photons [36], muons [37].

²In fact, if $E^2 \neq p^2 + m^2$, from $\mathbf{v} = \partial E / \partial \mathbf{p}$ we have not $\mathbf{v} = \mathbf{p}/E$. Thus, in four-dimensional notation, we have $v^\mu \neq p^\mu/m$ and, time-differentiating both sides, $a^\mu \neq F^\mu/m$ ($F^\mu = \dot{p}^\mu$ indicating the 4-force): Newton’s Law does not hold anymore.

Consequences on the absorption and the spectrum of gamma rays and on the synchrotron radiation have been investigated in noncommutative QED [38] as well.

Allen and Yokoo [39] list a series of proposed or performed experiments, on Earth and in space, to test both Lorentz and CPT symmetries: atomic experiments [10,3,32] (penning trap experiments with electrons, protons and their respective antiparticles, clock comparison experiments exploiting Zeeman and hyperfine transitions, spin polarized torsion pendulum experiments, etc.); clock-based experiments [32] to probe effects of variations in both orientation and velocity (employing H-masers, laser-cooled Cs and Rb clocks, dual nuclear Zeeman He-Xe masers, superconducting microwave cavity oscillators); experiments involving neutrino and kaon oscillations [32]; measurements of cosmological birefringence by interferometric searches of spacetime metric fluctuations [40].

Breakdown of the relativistic invariance and deformed “mass shell” relation have been advanced, above all, for *neutrinos* because of their very small mass and very large momenta. Consequences of a nonstandard dispersion law on neutrino oscillations has been analyzed in [7] and [14]; the observable effects of the non-Lorentz nature of the flavour eigenstates have been investigated by Blasone *et al.* [41], by Klinkhamer [42] and by Smolin and Magueijo [19]. In particular, striking effects on neutrino physics are expected [10,14,21,43] as a possible explanation of the arrival delays of neutrinos emitted from supernova SN1987A, as well as a possible kinematical stability of neutrons and pions of very high momentum. As stated in [18,21,28] SN1987A might constitute an interesting laboratory for studying LV’s, because of the relatively high energy of the observed neutrinos (up to 100MeV), the relatively large distances travelled (about 10^4 light-years), and the short (of the order of second) duration of the bursts.

By assuming the dispersion relation (7) for the mass eigenstates of electron neutrinos, Carmona and Cortes [21] succeed in explaining the so-called “tritium beta-decay anomaly”, i.e., the anomalous excess of decay events near the endpoint of the electron energy spectrum (where nonrelativistic few-eV neutrinos are produced) which yields a characteristic “tail” in the Kurie plot. If the Special Relativity dispersion law holds, this unexpected phenomenon involves *negative* squared masses for electron neutrinos. Instead, using eq.(7), those authors show that the excess of electron events is only apparent and no tachyonic neutrino is needed. In the same paper the Lorentz violating momentum-energy relation is exploited to account for the spread of arrival times of neutrinos from SN1987A, and for the stability of very high energy neutrons and pions in cosmic rays, since the high momentum decay reactions $n \rightarrow p + e^- + \bar{\nu}_e$, $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ become forbidden.

In the next sections we shall deduce upper bounds on the LV parameters appearing in equations (3) and (7), respectively, by using the available precision experimental data on the neutron lifetime and the decay rates of the charged π meson.

We assume that even if the kinematics of the above decays is affected by the LV in the dispersion law, nevertheless the dynamics, at the low energy scales of laboratory experiments, is essentially the one given by the Standard Model. If other radiative contributions (as additional Feynman diagrams) should appear as a consequence of the modified mass shell, we expected they are irrelevant at the considered energy scales, due to the excellent experimental verification of the Standard Model predictions at these scales.

II. NEUTRON β DECAY

Let us first consider modifications induced by a Lorentz violating dispersion relation for neutrinos on the lifetime of the neutron. The relevant decay channel is:

$$n \rightarrow p + e^- + \bar{\nu}_e , \quad (8)$$

and the differential decay rate, assuming that the Standard Model gives the dominant contribution, in the Born approximation can be written as [44]:

$$d\Gamma = \frac{eG_F^2(C_V^2 + 3C_A^2)}{(2\pi)^4} \delta(E_\nu - Q)p_\nu^2 dp_\nu p_e^2 dp_e d\Omega_e , \quad (9)$$

where G_F is the Fermi coupling constant and C_V , C_A are the vector and axial couplings. We have denoted with E_i, p_i the energy and 3-momentum of the particle i , while $Q = \Delta - E_e$ and Δ is the neutron-proton mass difference: $\Delta = M_n - M_p$. As a first case, we consider a neutrino dispersion relation of the form (7)

$$E_\nu^2 = p_\nu^2 + m_\nu^2 + 2\mu p_\nu . \quad (10)$$

In the ultrarelativistic limit $m_\nu \ll p_\nu$ (and also assuming that μ is small compared to the neutrino momentum) we thus have:

$$E_\nu \simeq p_\nu + \frac{m_\nu^2}{2p_\nu} + \mu. \quad (11)$$

For simplicity, in what follows we take $m_\nu = 0$; by inserting (11) into (9) and performing some integrations, at first order in small quantities we obtain:

$$\Gamma \simeq \Gamma_B + 2 \frac{\mu_e G_F^2 (C_V^2 + 3C_A^2)}{m_e (2\pi)^3} m_e^5 \int_1^{\Delta/m_e} d\varepsilon \varepsilon \sqrt{\varepsilon^2 - 1} \left(\varepsilon - \frac{\Delta}{m_e} \right), \quad (12)$$

where $\varepsilon = E_e/m_e$, and

$$\Gamma_B = \frac{G_F^2 (C_V^2 + 3C_A^2)}{(2\pi)^3} m_e^5 \int_1^{\Delta/m_e} d\varepsilon \varepsilon \sqrt{\varepsilon^2 - 1} \left(\varepsilon - \frac{\Delta}{m_e} \right)^2, \quad (13)$$

is the standard Born rate for the neutron decay. An accurate comparison with the experimental date, in order to get a constraint on the parameter μ_e , should require the computation of the radiative corrections (and similar ones) to the Born expressions (see, for example, [45]). However, here and in the following we are interested only in giving an order of magnitude estimate for μ_e , as can be deduced from laboratory experiments. To this end, let us write the total (corrected) neutron decay rate as:

$$\Gamma = \Gamma_0 + 2 \frac{\mu_e}{m_e} \Gamma_1, \quad (14)$$

where Γ_0 is the (corrected) Standard Model rate while $2m_e\Gamma_1/\mu_e$ represents the Lorentz-breaking term. The parameter μ_e is then obtained from:

$$2 \frac{\mu_e}{m_e} = \frac{\Delta\Gamma}{\Gamma_1} = \frac{\Delta\Gamma}{\Gamma_0} \frac{\Gamma_0}{\Gamma_1}, \quad (15)$$

$\Delta\Gamma = \Gamma - \Gamma_0$. The factor $\Delta\Gamma/\Gamma_0$ is approximately given by the relative uncertainty in the experimental determination of the neutron lifetime τ ; from the Particle Data Group analysis we deduce [46]:

$$\frac{\Delta\Gamma}{\Gamma_0} \leq \frac{\Delta\tau}{\tau} \sim 10^{-3}. \quad (16)$$

Instead, the factor Γ_0/Γ_1 can be roughly estimated as follows. From Eqs. (12) and (13) we can deduce that

$$\frac{\Gamma_0}{\Gamma_1} \sim \left\langle \left| \varepsilon - \frac{\Delta}{m_e} \right| \right\rangle, \quad (17)$$

where $\langle \varepsilon - \Delta/m_e \rangle$ denotes a sort of average over the electron energy spectrum. The kinematics of the neutron beta decay predicts that a typical average value for the electron energy is of the order of 1 KeV, so that we can estimate:

$$\frac{\Gamma_0}{\Gamma_1} \sim 2, \quad (18)$$

that is, such a value is of the order of unity. From Eq. (15) we finally obtain the following rough limit on the Lorentz-violating μ_e parameter for the electron-neutrino relation:

$$\mu_e \leq 1 \text{ keV}. \quad (19)$$

Similar considerations hold when we consider the dispersion relation in (3) and replace Eq.(10) with the following

$$E_\nu^2 = p_\nu^2 + m_\nu^2 + \alpha \frac{p_\nu^3}{M}. \quad (20)$$

Repeating the computations for the decay rate along the lines outlined above, Eq.(12) is now replaced by

$$\Gamma \simeq \Gamma_B + 2 \frac{m_e \alpha_e}{M} \frac{G_F^2 (C_V^2 + 3C_A^2)}{(2\pi)^3} m_e^5 \int_1^{\Delta/m_e} d\varepsilon \varepsilon \sqrt{\varepsilon^2 - 1} \left(\varepsilon - \frac{\Delta}{m_e} \right)^3. \quad (21)$$

Now we set:

$$\Gamma = \Gamma_0 + 2 \frac{\alpha_e}{M} m_e \Gamma_1, \quad (22)$$

and

$$\frac{\Gamma_0}{\Gamma_1} \sim \left\langle \left| \varepsilon - \frac{\Delta}{m_e} \right| \right\rangle^{-1}. \quad (23)$$

We thus obtain the following order of magnitude constraint on the ν_e parameter α_e :

$$\alpha_e^{-1} M \geq 1 \text{ GeV}. \quad (24)$$

III. PION LEPTONIC DECAY

Since long time the ratio of the decay rates of the charged π meson,

$$R_{e\mu} = \frac{\Gamma(\pi \rightarrow e\nu_e)}{\Gamma(\pi \rightarrow \mu\nu_\mu)}, \quad (25)$$

has been used to test fundamental properties of elementary particle physics; this quantity in fact at a first, good approximation, is independent of the details of π meson interactions [44]. Assuming that the dynamics of the considered decays is substantially described in the framework of the Standard Model, the ratio in (25) can also be useful to obtain constraints on the Lorentz-breaking parameters in the neutrino dispersion relations. Note that, on the contrary to what happens for the neutron decay, the single decay rates (instead of their ratio) cannot give useful informations on these parameters, since the experiments measuring directly the above mentioned processes are used to fix the value of the pion decay constant f_π [46]. The decay rate of a charged pion into a lepton-neutrino pair can be written as [44]:

$$\Gamma(\pi \rightarrow l\nu) = \frac{G_F^2 \cos \theta_c^2 f_\pi^2 m_l^2 E_\nu}{2\pi} \int \frac{p_\nu^2 dp_\nu}{E_\nu E_l} \delta(E_\nu + E_l - m_\pi). \quad (26)$$

where θ_c is the Cabibbo angle. Assuming the dispersion relation in Eq. (11) (with $m_\nu = 0$) for neutrinos, the integral in Eq. (26) evaluates to:

$$\frac{E_\nu}{E_\nu + E_l} - \mu \frac{E_\nu + 2E_l}{(E_\nu + E_l)^2} = \frac{m_\pi^2 - m_l^2}{2m_\pi^2} - \mu \frac{3m_\pi^2 + m_l^2}{2m_\pi^3}. \quad (27)$$

Thus, at the first order in small quantities, the decay rate takes the form:

$$\Gamma(\pi \rightarrow l\nu) \simeq \Gamma_B \left\{ 1 - 2 \frac{\mu}{m_\pi} \frac{m_\pi^2 + m_l^2}{m_\pi^2 - m_l^2} \right\} \quad (28)$$

where

$$\Gamma_B = \frac{G_F^2 \cos \theta_c^2 f_\pi^2 m_l^2 m_\pi}{8\pi} \left(1 - \frac{m_l^2}{m_\pi^2} \right) \quad (29)$$

indicated the standard Born rate [44]. A precise determination of the μ parameter again would require the computation of the radiative corrections (see for example [47]) but, as in the neutron decay case, we are interested only in an order of magnitude estimate, so that we will now proceed as in the previous section. The ratio in Eq. (25) is, then, written as follows:

$$R_{e\mu} \simeq R_{e\mu}^0 \left\{ 1 - 2 \frac{\mu_e}{m_\pi} \frac{m_\pi^2 + m_e^2}{m_\pi^2 - m_e^2} + 2 \frac{\mu_\mu}{m_\pi} \frac{m_\pi^2 + m_\mu^2}{m_\pi^2 - m_\mu^2} + \text{radiative corrections} \right\}, \quad (30)$$

where

$$R_{e\mu}^0 = \frac{m_e^2}{m_\mu^2} \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 - m_\mu^2} \quad (31)$$

is the standard value of the ratio in (30). From the experimental values for the particle masses [46] and the ratio $R_{e\mu}$, we deduce the following constraint on the ν_e and ν_μ Lorentz-violating parameter:

$$-1.0\mu_e + 3.7\mu_\mu \leq 0.2 \text{ MeV}. \quad (32)$$

The evaluation of the rate in the case of the modified dispersion relation in (20) proceeds in an analogous way; after some algebra we obtain:

$$\Gamma(\pi \rightarrow l\nu) \simeq \Gamma_B \left\{ 1 - \frac{\alpha m_\pi}{8M} \left(1 - \frac{m_l^2}{m_\pi^2} \right) \left(3 + 5 \frac{m_l^2}{m_\pi^2} \right) \right\}; \quad (33)$$

then the relation in Eq. (25) takes the form:

$$R_{e\mu} \simeq R_{e\mu}^0 \left\{ 1 - \frac{\alpha_e m_\pi}{8M} \left(1 - \frac{m_e^2}{m_\pi^2} \right) \left(3 + 5 \frac{m_e^2}{m_\pi^2} \right) + \frac{\alpha_\mu m_\pi}{8M} \left(1 - \frac{m_\mu^2}{m_\pi^2} \right) \left(3 + 5 \frac{m_\mu^2}{m_\pi^2} \right) + \text{radiative corrections} \right\}. \quad (34)$$

By using the experimental date [46] we have:

$$-3.0 \frac{\alpha_e}{M} + 2.5 \frac{\alpha_\mu}{M} \leq 186 \text{ MeV}^{-1}. \quad (35)$$

IV. CONCLUSIONS

A possible violation of Lorentz symmetry can manifest into a variety of process, briefly outlined in the introduction. However, the energy scales relevant for these processes are usually extremely large, and the corresponding phenomenology is, consequently, not accessible by laboratory experiments, so that only very poor limits on Lorentz violating parameters can be obtained. A slightly better situation arises in neutrino phenomenology where, on one hand, some parameters are completely unconstrained and, on the other hand, very precise experiments are available. In this paper we have determined the actually most stringent limits on several Lorentz violating parameters in the neutrino sector coming from laboratory experiments. In particular we have focused on the modifications induced by Lorentz violation on the energy-momentum dispersion relations for ν_e and ν_μ in two typical cases considered in many different models appeared in the literature. We have thus reviewed the existing experiments and found that the most stringent bound on ν_e parameters comes from the neutron β decay process, while constraints on ν_μ parameters have been obtained by analyzing pion leptonic decays. The results obtained have been shown in Sections 2 and 3. Those two particular cases of the LV dispersion relation (linear and cubic corrections) which we have here studied constitute a starting point for a more general phenomenological analysis. Actually, a complete theory accounting for Lorentz violation in the neutrino sector, is the Standard Model Extension developed in [48]. The most general form of the energy-momentum dispersion law for neutrinos, including all the violating constants, is obtained in this framework (see, for example, Eq. (2) in the first paper of Ref. [48]). A comprehensive analysis of laboratory bounds on these parameters will be performed in a forthcoming paper. However, although the expected parameter region should be, as deduced in the present work, only poorly constrained, nevertheless the bounds considered here for the first time indicate the sensitivity of terrestrial experiments to this kind of physical problem, and set the starting point for future experimental investigations on Lorentz symmetry violation.

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